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REVIEW

Estimation of Future Mortality Rates and Life Expectancy in Chronic Medical Conditions

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Estimates of old-age mortality are necessary for the construction of life tables and computation of life expectancy, and are essential in the growing area of life insurance for the elderly. Two common assumptions are that either the excess death rate (EDR) or the relative risk (RR) stays constant with increasing age. It is known, however, that for most medical conditions the former underestimates the risk and the latter overestimates it. A third popular method is that of rating up: a subject is said to be ''rated up k years'' if his future mortality rates are assumed to be those of a person in the general population who is k years older. It is shown here that this method generally leads to gross overestimates of old-age mortality.

We consider two less-commonly used models, log-linear declining relative risk (LDR) and constant proportional life expectancy (PLE), and compare them to the methods of constant EDR, constant RR and rating up. Although slightly more complicated to employ than the other methods, both LDR and PLE generally give better estimates of mortality and life expectancy.

When mortality rates for chronic conditions are known within a certain age range, and estimates outside of the range are required, the LDR and PLE methods may be preferable to the more familiar methods of constant EDR, constant RR, or rating up.

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True mortality rates for many conditions, such as cerebral palsy or spinal cord injury, are known with accuracy only up to certain ages, usually around 70.[1–4] The rates at higher ages are often either unknown or unreliable, being based on small samples. For many purposes, however, mortality rates are required at all ages. Important examples are the estimation of life expectancy, valuation of life annuities, life insurance policies for the elderly, and litigation involving wrongful death or lifetime cost of care.[5] A complete schedule of mortality rates requires the use of methods, such as the ones explored here, to estimate mortality rates for old ages.

The objective of this study is to compare five methods of estimation of mortality rates at advanced ages. It may be noted that the methods also can be used to extrapolate to earlier ages. For example, one could assume that data on older persons may be applied to younger persons.

In this paper, we are concerned with chronic conditions, such as cerebral palsy or spinal cord injury, rather than with acute or progressive conditions, such as cancer, HIV or

Parkinson disease. The mortality in the latter may be quite different than would be predicted by the models considered here. In cancer, for example, the excess death rate generally declines with increasing time from diagnosis or treatment, a pattern incompatible with any of the methods considered here.

Three well-known methods—the assumptions of constant excess death rate (EDR), constant relative risk (RR) and rating up—are simple to apply but generally lead to biased estimates of mortality in old age. Constant excess risk systematically overestimates life expectancy, while constant relative risk and rating up systematically underestimate it. As a result we consider other methods here. We focus on two less well-known methods, those of log-linear declining relative risk (LDR) and proportional life expectancy (PLE). We compare the results of the five methods using data where the true mortality rates are known with some precision.

THE FIVE METHODS

We use standard notation for quantities related to the survival distribution. Let T be the individual's survival time, and t denotes a specific value of the (stochastic) variable T. Let F(t) be the cumulative distribution function: $F(t) = P(T \leq t)$, the probability that the survival time is $\leq t$. Let f(t) be the probability density function, and $S(t) = 1 - F(t)$ be the survival function, ie, the probability that the individual is alive at time t. The mortality rate (or hazard rate) is $h(t) = f(t)/S(t) = d/dt\{-\ln S(t)\}$.

Provided they are constant over the 1-year or 5-year periods considered, these age-specific mortality rates are precisely those given in the column labeled ''m(t)'' of a life table.

The life expectancy at time t, denoted here by e(t), is the expected (ie, average) remaining survival time for an individual alive at time t: $e(t) = \int tf(t)\,dt$, where the integral is over all subsequent ages. It is known[6,7] that e(t) can also be expressed as $e(t) = \int S_t(x)\,dx$, where $S_t(x) = P(T > x \mid T > t)$, for $x \geq t$, is the conditional survival function given survival to time t.

These quantities are taken to apply to survival in a reference population, often simply the general population. We are also concerned with survival of persons with a given medical condition and use the subscript c to distinguish this.

We now examine the assumptions and specifications of the five models.

Constant Excess Death Rate

The assumption of the constant excess death rate (EDR) method is that the mortality rate $h_c(t)$ of the population of interest exceeds h(t), the mortality in the reference population, by a constant.[8(p31),9] That is, $h_c(t) = h(t) + k$, where the EDR, k, is the same at every age. For many medical conditions, good data is available on the EDRs for ages up to about 70. If one is willing to assume that the EDR remains constant at older ages, the rates in the target population, $h_c(t)$, are immediately determined, and these can be used to construct a complete life table. The simplicity of this method is attractive, and it is widely used.[10,11] However, as we shall see, empirical data indicate that the assumption of constant EDR generally underestimates $h_c(t)$ at old ages. Hence, this is one reason for interest in other assumptions, even at the cost of greater complexity.

Constant Relative Risk

The assumption of the constant relative risk (RR) method is that the mortality rates of the population with the disease and of the reference population have a constant ratio over time: $h_c(t) = k \times h(t)$, where k does not depend on t. This assumption is implicit in many studies that work with RR or mortality ratios (MR). It is widely used in the fields of underwriting,[12,13] epidemiology, and actuarial science.[8(p67)] Medical risk evaluators also often use this method. For example, a rating of +50 suggests that the age-specific mortality rate should be augmented by 50% at all ages of

interest, which corresponds to a constant relative risk of 1.5.

As in the case of the constant EDR method, the simplicity of calculations is accompanied by a serious bias, at least for long-term projections: constant RR leads to mortality rates that are generally too high at older ages. This is well documented in the literature.[8]

Rating Up

In this method, one advances the individual's age by a suitable number of years, k, and then assumes the subsequent mortality rates to be the reference population rates for a person k years older. For example, a 40-year-old smoker may be considered comparable to a person of age 45 in the general population. In this case, he would be said to have been ''rated up'' by k = 5 years. One then assumes that at age 62 our individual will have the mortality rate of a normal 67 year old. The assumption here is that $h_c(t) = h(t + k)$.

The quantity k is sometimes chosen so that the individual's (known) hazard at age t matches the reference population rate at age t + k.

This method is well known to underwriters and has been recommended for medicolegal application[14(§26)] and certain medical conditions.[12] As in the cases of constant EDR or RR, it has the appeal of simplicity: it requires neither new calculations nor the construction of a new life table. Again, however, we shall see that this comes at the cost of a serious bias.

Note: The term rating up is sometimes used in a different sense, in connection with life expectancy. Suppose, for example, that it is known or estimated that a given 8-year-old boy with severe cerebral palsy has a life expectancy of 15 years. As a normal male of age 66 in the general population has approximately this same life expectancy, one could assume that the boy has the same schedule of future mortality rates as the 66-year-old man. This method again is convenient to use—one can work solely with standard life tables—and obviously ensures that the correct (or assumed) life expectancy is preserved. A problem, however, is that it gives the wrong pattern of mortality rates: while the hazard rates for a child with severe cerebral palsy are fairly constant over time,[15] those for a man of age 66 are initially lower and then climb rapidly with age. We emphasize that rating up in this second sense is not considered in the present article.

Proportional Life Expectancy

Anderson[8,16,27] found empirically that for certain medical conditions, the proportional life expectancy (PLE) remains approximately constant at all ages. For example, life expectancy in a given condition may be 60% of normal, whether the individual is 40, 60 or 80 years old. This is the assumption of PLE. Mathematically, the assumption is: $e_c(t)/e(t) = r$, where r is a constant independent of the age t.

At first sight, it may seem that even if this were true it would be difficult to apply in practice because the relationship between h(t) and $h_c(t)$ that it implies is not at all obvious. However, the relationship proves to be remarkably simple: *if PLE holds, then the excess death rate at any age is inversely proportional to the remaining life expectancy at that age.* A mathematical proof of this assertion is given in the Appendix.

To illustrate this ''PLE'' method, suppose the EDR for a male with a given condition at age 60 is known empirically to be 0.010 (ie, 10 deaths per 1000 person-years), and we wish to estimate the EDR at age 80. The general population male life expectancies at ages 60 and 80 are 19.4 and 7.5, respectively.[17] According to PLE, therefore, the EDR at age 80 is $0.010 \times 19.4/7.5 = 0.026$.

By making such calculations at all ages, one can compute a complete schedule of mortality rates and thus a life table.

Because life expectancies generally decrease with age, the excess death rates predicted by this method will increase with age. PLE, therefore, yields higher mortality rates and lower life expectancies than the constant

EDR method. On the other hand, PLE results in lower mortality rates than the assumption of constant relative risk (because life expectancy at older ages declines more slowly than the mortality rates increase). This comports with Singer's observation that for many medical conditions, the mortality ratios decrease with age, while the excess death rates increase.[18(p18)] Therefore, PLE occupies a middle ground between constant RR (which tends to underestimate life expectancy) and constant EDR (which tends to overestimate it). As we shall see, in many cases PLE gives better estimates of old-age mortality than either.

Anderson[8,16] has pointed out that in some conditions the proportional life expectancy varies slowly and in a linear fashion rather than remaining constant as assumed here. By modeling this linear trend one could in principle arrive at an improved version of the PLE method. However, such a method would be extremely complex to apply in practice for two reasons. First, the rate of linear change would have to be estimated, which is a difficult task in most cases. Secondly and more seriously, if this more general model is assumed, there appears to be no simple mathematical relationship between the required hazards $h_c(t)$ and the known hazards $h(t)$.

Log-Linear Declining Relative Risk

In many populations (including the general population), age-specific mortality over the age-range of 30 to 75, say, roughly follows the Gompertz Law[19] $h(t) = \exp(\gamma + kt)$, for suitable constants γ and k. That is, mortality grows exponentially with age. If such a relationship holds for both the general population and the population with the medical condition, then it follows that $\ln\{h_c(t)\} = \ln\{h(t)\} + \beta(\alpha - t)$ for some constants α and β, and a suitable range of ages t. An equivalent expression is: $\ln[h_c(t)/h(t)] = \beta(\alpha - t)$, which shows that the logarithm of the relative risk declines linearly with increasing age.[1] We refer to this model as log-linear declining risk (LDR). Note that the Gompertz law is sufficient, though not necessary, for LDR to hold.

At age $t = \alpha$ the hazards rates $h_c(t)$ of persons with the condition and $h(t)$ in the reference population become equal. This age is termed the parity age. We have found in practice that the parity age is at least 100.[1] Beyond that age, the choice of h_c has little bearing on life expectancy, and our suggestion is to use $h_c(t) = h(t)$ for $t > \alpha$.

Although in certain cases we consider parity ages well in excess of 100 (the assumption of constant RR is equivalent to an infinite parity age), we do not assert that mortality rates beyond age 100 really follow the above law—observations beyond age 100 are too uncommon for this to be either testable or important in practice. Instead, the assertion is only that mortality rates at younger ages are consistent with a law that gives parity at the given age.

Because the mortality rates $h(t)$ are known from standard sources,[1] once α and β are known the mortality rates $h_c(t)$ can be computed directly. In practice the parity age α has in some cases been estimated from previous research (an estimate of 100 years is reasonable in many instances), and the parameter β is determined if, as is usually the case, the relative risk is known at some given age. For example, if the logarithm of the relative risk is known to be 4.0 at age 30, and is assumed to be 0 at age 100 (which is therefore the parity age), then at the halfway point, age 65, the logarithm of the relative risk is estimated to be 2.0. That is, the hazard rate for the condition at age 65 is taken to be the general population rate multiplied by $e^{2.0}$ (where $e = 2.718$ is the base for natural logarithms). In this example, $\beta = [4 - \ln(1.0)]/(100 - 30) = 0.057$.

With increasing age, conditions such as heart disease or cancer become frequent in the general population. As the death rate from such conditions may be no higher among persons with the medical condition in question, the result is usually that relative risks decline sharply with age.[1] The LDR method appropriately constrains the relative risks to decrease with age, rather than to stay constant. In practice, LDR—like PLE—occu-

pies a middle ground between the generally biased methods of constant EDR and constant RR.

EXAMPLES

To test the accuracy of the five methods, we applied the procedures to several conditions where the true mortality rates at advanced ages were known with some accuracy. Although in the previous section we worked with continuous time, the examples that follow are necessarily based on a ''discrete'' time scale—either 1-year or 5-year intervals.

Sex Differences

Though not a medical condition, male sex is associated with higher mortality. It is a useful example for our purposes because the ''effect'' of male sex (eg, its EDR or RR) is known with precision from standard tables. Here we used mortality rates based on data from Finnish males and females, 1990 to 1995. The male mortality rates were known for all ages,[20] but for the present exercise we assumed that male mortality was known only up to age 30 (we refer to age 30 here as the anchor age) and had to be estimated from females rates thereafter, using one of the five methods.

To illustrate the calculations for the five methods, the following facts are needed:

Mortality rate of females at age 30	0.000449
Mortality rate of males at anchor age 30	0.001578
EDR for males at age 30	0.001129
RR for males at age 30	3.5
Rating up (male mortality at age 30 = female mortality at age 43)	+13 years
Life expectancy of females at age 30	50.43

Based on the above, we can estimate the male mortality rates for ages beyond 30. We illustrate this for age 50; the calculations for other ages are analogous. The base female mortality rate at age 50 is 0.002513.

a) Constant excess death rate: the estimated male mortality rate at age 50 is 0.002513 + 0.001129 = 0.003642.

b) Constant relative risk: the estimated male rate at 50 is 0.002513 × 3.5 = 0.008796.

c) Rating up does not require any computation; we estimate the male rate at age 50 to be that of a 63-year-old (=50 + 13) female. This is 0.007923.

   Note: Because published mortality rates generally do not extent beyond age 109, and in some cases not beyond age 100, there is an upper limit to the age at which the rating up method may in practice be applied. This is reflected in the Figures presented here. Whereas the graphs for the other methods extend to age 100, those for rating up end at earlier ages.

d) The PLE method requires the life expectancies in the reference population. Here, the life expectancy for females at age 50 is 31.4 years. From this, and the life expectancy of females at age 30 (50.4 years), we estimate the excess death rate at age 50 to be the ratio of life expectancies in the reference (here female) population multiplied by the excess death rate at age 30. That is, the EDR for males at age 50 is the EDR at 30 times the ratio of life expectancies at 30 and 50: 0.001129 × 50.4/31.4 = 0.001812.

   Finally, the estimated male rate at age 50 is the female rate at 50 plus this EDR: 0.002513 + 0.001812 = 0.004325.

e) The LDR method requires two steps. The first is to find the rate of the annual (linear) decrease in the log of the relative risk. We assume here that the relative risk will reach 1 (and thus its logarithm will reach 0) at age 100. It follows that with each year of age from 30 to 100 the log of the relative risk will decrease by: $\beta = [\ln(3.5) - \ln(1.0)]/[100 - 30] = 0.01790$. This is the β of the previous section, section ''Log-linear Declining Risk.''

   The second step is to find the estimated relative risk at age 50 and apply it to the female rate at age 50. The log of this relative risk is equal to $\ln(3.5) - [(50 - 30) \times 0.01790] = 0.8948$.

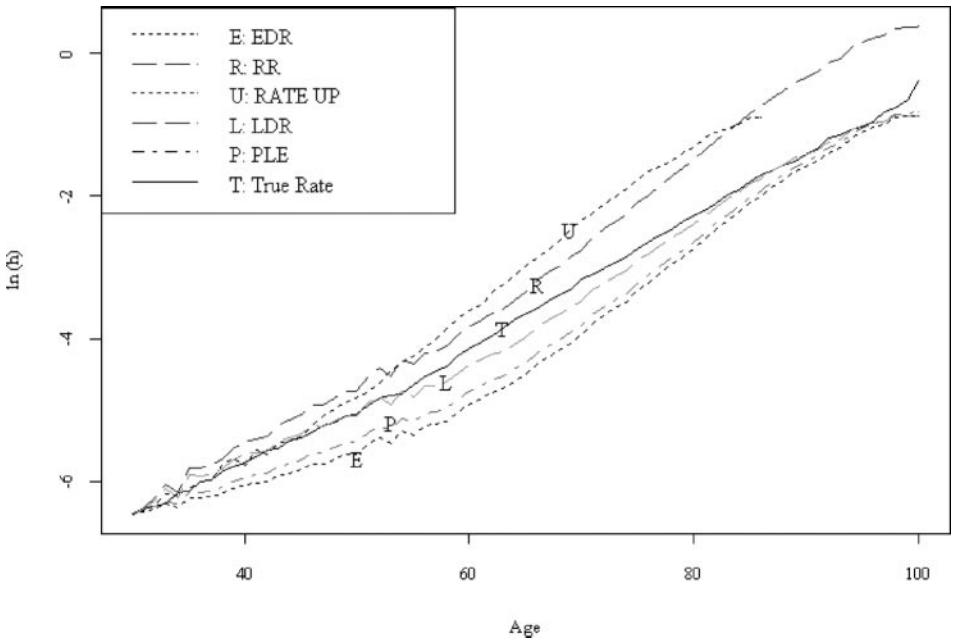

Figure 1. *Natural log of mortality rate by age: sex difference.*

That is, the log of the projected relative risk is smaller than the log of relative risk at age 30 by a total of 20 increments of the annual decline (β). This gives an estimated relative risk of exp{0.8948} = 2.447, and therefore an estimated mortality rate at 50 of 2.447 × 0.002513 = 0.006149.

Computations for all ages are shown in Figure 1. The figure shows that the rating up method grossly overestimates the true mortality rates. Also, as expected, the constant RR and constant EDR methods respectively overestimate and underestimate the true rates, the former dramatically so at old ages. Overall, LDR appears to be the most accurate and PLE is the second best, although in this case PLE tends to underestimate the mortality rates.

Note that the results are dependent on the ''anchoring'' age, in this case 30, and on the actual difference in mortality at that age [$h_c(t) - h(t)$], as this difference serves as the basis for all the estimated results.

Era Differences

As a second example, we used the mortality rates of one period to predict those of another. Era is of course not *per se* a medical condition; it is used here nonetheless because again the true mortality rates are known with great precision over the whole age range. We illustrate with the example of mortality rates of Japanese females for two periods, 1970 to 1975 and 1950 to 1955, to which we will refer here as late and early.[20] As were the male rates in the previous example, the early rates were assumed unknown for ages 30 and above. The five methods were then used to estimate early rates for ages 31 to 100 based on the late rates (Figure 2).

The pattern in Figure 2 is similar to the previous one in that rating up grossly overestimates the rates, constant RR overestimates them to a somewhat lesser extent, and constant EDR underestimates them. LDR and PLE give the best estimates, although the former overestimates rates at younger ages.

Diabetes

Data for diabetes mellitus were extracted from Gu et al,[21] which in turn was based on the First National Health and Nutrition Examination Survey of 14,374 subjects in 1971–1975. These authors assessed deaths through 1993, and computed mortality rates for quinquennial age groups, 30–35 to 85–90. Here the data were presumed unknown for ages 33

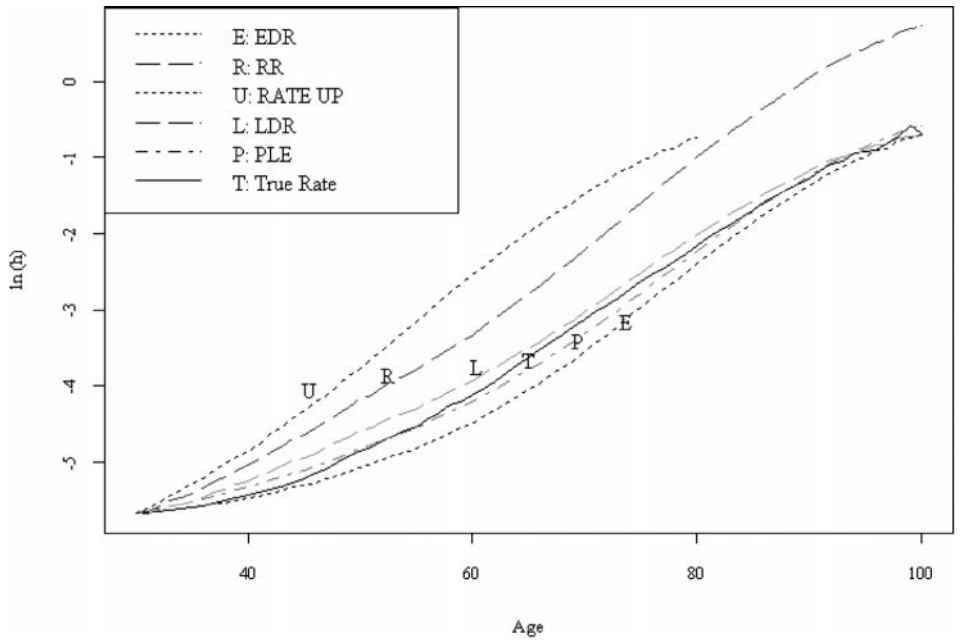

Figure 2. *Natural log of mortality rate by age: era difference.*

and above, and the five methods were then used to estimate these.

Unlike cerebral palsy and spinal cord injury, diabetes mellitus is not a static condition, and further the mix of Types I and II diabetes, each of which has its own pattern of mortality, may well vary with age. Nevertheless, it may be of interest to see how the various methods predict aggregate diabetes mortality at older ages. An advantage of diabetes as an example here is that it is a very common condition, with relatively good ''truth'' data on mortality rates over a broad age range.

As the true mortality data for diabetes were available in 5-year age groups, these were plotted as discrete points rather than as a continuous curve (Figure 3).

Once again, rating up grossly overestimates mortality rates and constant RR overestimates them to a smaller extent. Constant EDR again underestimates the risks at younger ages. Overall, LDR estimates are the most satisfactory, and the PLE method is second best, although it clearly underestimates the risks up to age about 70 years.

Cerebral Palsy

Data from California were used here.[1,15,22] Subjects had cerebral palsy and could not crawl, creep, scoot, walk or feed themselves. Mortality rates were obtained for quinquennial age groups 30 to 70. The true rates were presumed unknown for ages 33 and above, and the five methods were used to estimate these. Estimated rates were then compared to the true rates (Figure 4).

The empirical mortality rates in 5-year age groups present a somewhat irregular pattern, presumably because of sampling variation in the smaller groups. Nevertheless, the familiar biases in the rating up and constant RR and EDR methods are observed. Again the LDR and PLE methods, which give rather similar estimates, appear to be best.

Spinal Cord Injury

Data for spinal cord injury (SCI) were from the National Model Systems Spinal Cord Injury data base as presented in Strauss, DeVivo and Shavelle (2000).[4(Tables 2 and 3)] Two subpopulations were examined: high tetraplegics (injuries at the level of one of the first four cervical nerves, C1 to C4) and low tetraplegics (C5–C8). Only the three most severe grades of injury (A, B and C) were considered here. We chose cervical injuries because of the high mortality rates, which enabled us to estimate the true rates over a wide age range

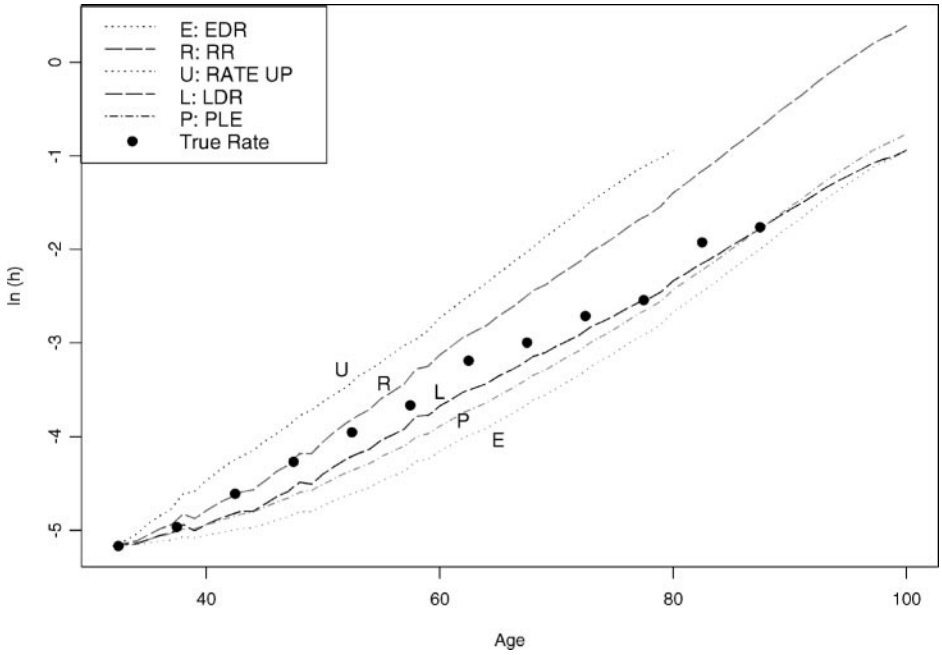

Figure 3. *Natural log of mortality rate by age: diabetes.*

with reasonable accuracy. Mortality rates were obtained for quinquennial age groups 30 to 70. Again, the true rates were presumed unknown for ages 33 and above, and the five methods were used to estimate these for ages 33 to 100 (Figure 5).

Parity ages in SCI for the LDR method had been previously considered by Bush et al.[23] Those authors found that the rate of decline in relative risk with age was slower than predicted from the ''usual'' parity age of 100. They suggested that an infinite parity age (ie, constant relative risk) was appropriate for C4–C8 grade A (complete) injuries, and that a parity age of 118 be used for all other combinations of level and grade. However, if a single parity age were to be used for all spinal cord injuries, the best estimate was 140. For simplicity we have used this last figure for all SCI related computations reported here.

The results for these spinal cord injury subgroups show a different pattern than that observed for the other conditions. Constant RR provides the best estimates up to the 50–55 age group. Thereafter, the method overestimates the mortality rates, as usual. LDR underestimates the true rates up to this point, though it gives good estimates thereafter. The PLE estimates are again similar to, but slightly lower than, those from LDR, and as usual the assumption of constant EDR underestimates the true rates at all ages.

The pattern for C5–C8 spinal cord injuries (Figure 6) is similar to that of C1–C4 injuries (Figure 5). Again, constant RR produces good results for younger ages but later tends to overestimate the rates. Here, too, LDR significantly underestimates rates for younger ages, but gives reasonable estimates at older ages. The fact that the data underlying Figures 5 and 6 are entirely separate but give similar results suggests that these results are not an artifact of sampling variation.

COMPARISON OF ESTIMATES

For many purposes, the life expectancy is an important summary measure of the schedule of mortality rates. In this section, we compare life expectancies resulting from the five methods with the ''true'' values of life expectancy.

The Table shows the results for the five conditions we considered in the previous section. Two sets of estimated rates were used for the Sex Differences and Era Differences, the first anchored at age 30 and the second at

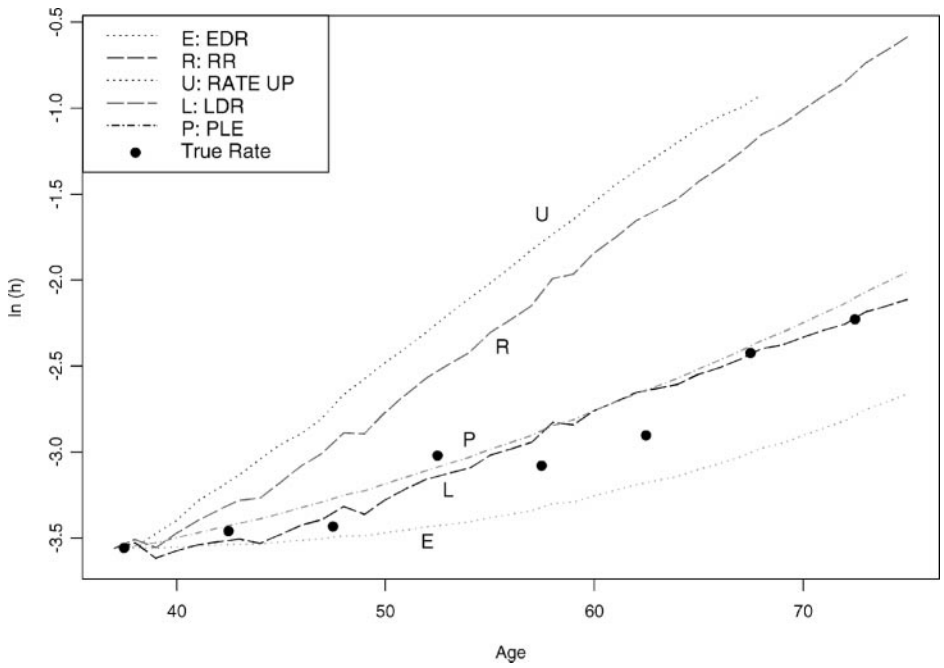

Figure 4. *Natural log of mortality rate by age: cerebral palsy.*

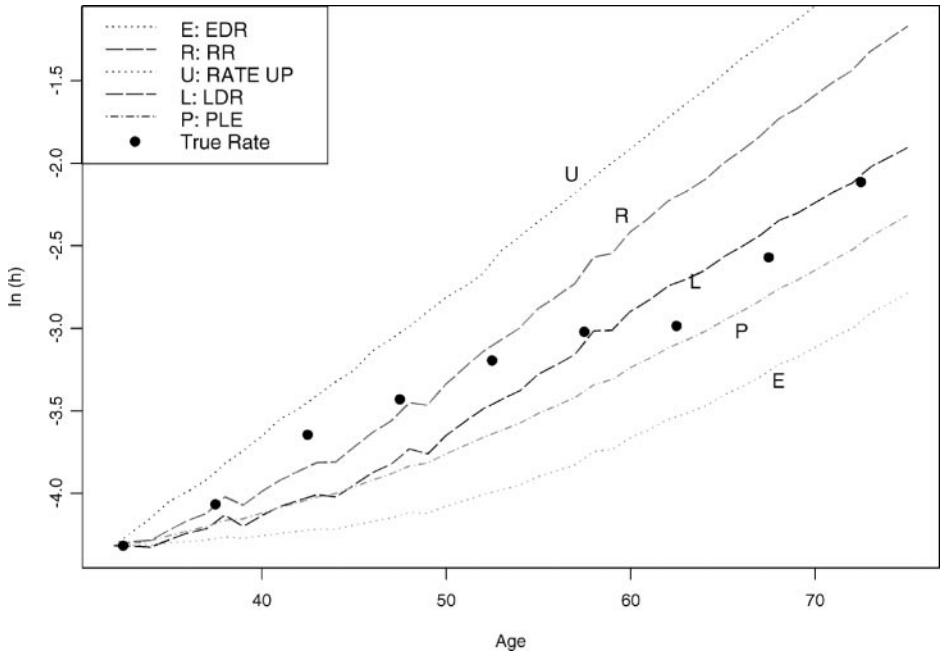

Figure 5. *Natural log of mortality rate by age: spinal cord injury, levels C1–C4.*

age 60. In both cases, life expectancies at age 60 and age 75 were also computed.

Life expectancies were calculated for each condition at the given ages based on six schedules of mortality rates: those estimated by the five methods, plus the true rates. The values in the Table show the difference between (a) the life expectancy as calculated from the estimated rates and (b) the true life expectancy. For each comparison, the best result is shown in bold. For example, the first row corresponds to the data of Figure 1. The true Finland male life expectancy at age 30 was 43.4 additional years.[20] The assumption of constant EDR for males (compared to females) seriously underestimates the difference between the sexes at older ages, and results in a 5.6-year overestimate of male life

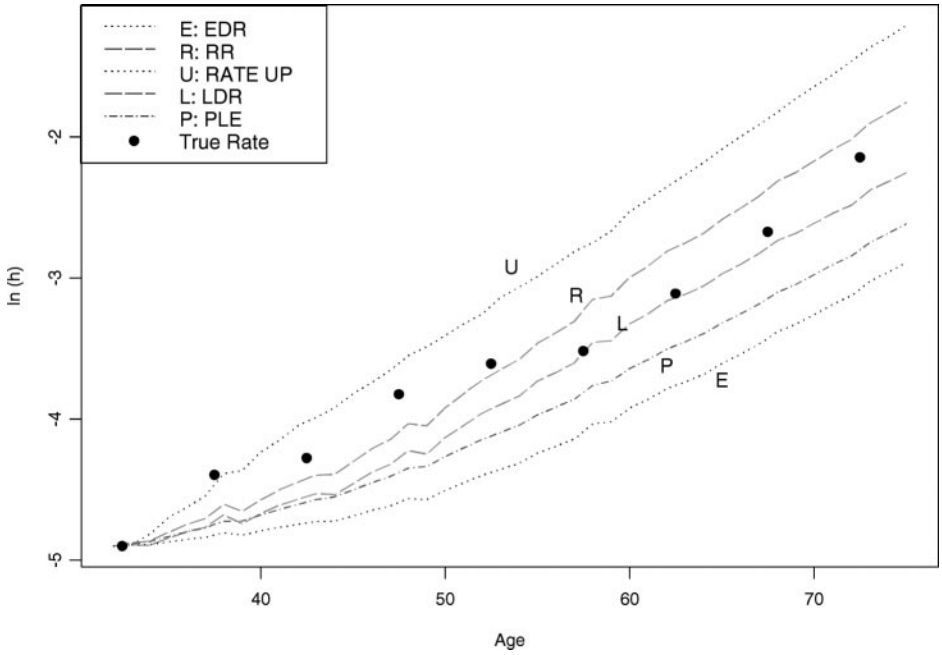

Figure 6. *Natural log of mortality rate by age: spinal cord injury, levels C5–C8.*

Table. Comparison of Life Expectancy Estimates From the Five Methods

	Anchoring Age	LE at age	True Value of LE	Estimate Minus True Value				
				EDR	RR	Rate Up	LDR	PLE
Sex Differences	30	30	**43.4**	5.6	−4.3	−5.5	**1.3**	4.4
		60	**17.6**	4.5	−3.7	−5.6	**1.4**	3.6
		75	**8.4**	2.2	−3.4	−4.1	**0.5**	1.7
	60	60	**17.6**	2.3	−1.7	−1.9	**0.3**	0.9
		75	**8.4**	1.6	−2.2	−2.2	**−0.1**	0.4
Era Differences	30	30	**41.8**	2.7	−7.5	−12.1	−1.8	**0.5**
		60	**17.0**	2.5	−6.1	−10.0	−1.0	**0.9**
		75	**7.8**	1.2	−4.2	−5.2	−0.6	**0.3**
	60	60	**17.0**	1.3	−1.5	−1.7	**−0.1**	0.4
		75	**7.8**	0.9	−1.7	−1.7	−0.2	**0.1**
Diabetes	32.5	30	**35.8**	7.0	**−2.8**	−6.5	**2.8**	4.3
Cerebral Palsy	37.5	30	**29.7**	2.3	−5.7	−7.6	**−1.1**	−1.2
Spinal Cord Injury Levels C1-C4, Grades ABC	32.5	30	**26.6**	8.1	**−1.4**	−5.1	1.9	4.0
Spinal Cord Injury Levels C5-C8, Grades ABC	32.5	30	**31.5**	8.2	**−0.1**	−4.3	2.8	5.4
Average absolute error (years)				5.7	3.6	6.9	**2.0**	3.3
Average absolute error (as % of true values)				16%	10%	20%	**6%**	10%
Average error (as % of true values)				16%	−10%	−20%	**3%**	8%

Note: Parity ages for LDR were all 100 except for spinal cord injury, where age 140 was used.[23]

expectancy. Conversely, constant RR leads to a 4.3-year underestimate, and rating up to a 5.5-year underestimate. The closest estimate is given by LDR, which overestimates by 1.3 years.

For Era Differences rather similar findings are obtained, although in this case PLE is slightly better than LDR. For diabetes, none of the methods estimates the life expectancy especially well, although LDR and constant RR give the best estimates. PLE and LDR are clearly the best methods for the case of cerebral palsy, giving errors of approximately 1 year only. Finally for spinal cord injury, the best estimates are obtained from constant RR, although LDR is a clear second.

At the foot of the Table, we present some summary measures of the overall performance of the five methods. The first two rows give the average over all the comparisons (at age 30 only) of the absolute error (ie, with signs ignored and absolute values) and of the errors expressed on a percentage basis. The third row is the average of the errors (expressed as a percentage of the true life expectancies). According to this, the best method is LDR, which on average overestimates life expectancies by 3%. The worst is rating up, which underestimates them by an average of 20%. PLE gives results rather similar to those of LDR, though tends to overestimate the true values by a slightly larger amount. Finally, constant EDR leads to serious overestimation of life expectancy, with an average error of 16%. In every example we considered, constant EDR overestimated the true life expectancy, and constant RR underestimated it.

DISCUSSION

In actuarial work pertaining to persons with chronic medical conditions, it is frequently necessary to extrapolate mortality rates beyond the age range where reliable data is available. This is important in the pricing of life annuities and life policies for the elderly, in medicolegal work requiring an estimate of the cost of lifetime care and in scientific work on the estimation of life expectancy.

We have seen that the commonly used method of constant relative risk generally overestimates mortality at old ages, and therefore underestimates life expectancy. The reason is that natural causes, such as heart disease and cancer, tend to be the main cause of death in old age, and these conditions may be nearly as common in the general population as in persons with the condition of interest. For example, the relative risk for a young person with severe cerebral palsy may be 100 or more,[15] but in old age, where the general population mortality rates eventually exceed 8% or 80 per 1000, such a relative risk is inconceivably high.

Our impression is that this aspect of the constant RR method is well known amongst life policy underwriters, who may feel that its use is justified because it is conservative. This is true for the writing of life policies, though of course the reverse is true for the pricing of single-premium annuities. However, a case can be made for the use of more accurate methods, such as those described here. One can still build conservatism into the pricing; the difference is that one will then know the added margin of error more precisely.

It may be appropriate to comment on the use of life expectancy as a criterion for comparison of the methods. Life expectancy is a useful summary measure of longevity in pricing of single-premium annuities, and for structured settlements involving disabilities arising from personal injuries. In life insurance underwriting, however, the probability of lapses are taken into account, and greater weight should be placed on short-term rather than on long-term mortality. Perhaps a criterion other than life expectancy may be more appropriate for this application. For short-term applications, mortality data for the age range of interest may exist. In such cases, the methods discussed here are not needed.

The tendency of relative risks to decline steadily with age creates a problem in many applications. If a published study shows that a given condition is associated with a relative

risk of 2 (ie, a rating of +100), it cannot be assumed that this applies at all ages. It is necessary to know the age range on which the figure is based. For example, a relative risk of 2 for a disease of childhood would have only a very small effect on life expectancy because childhood mortality is so low. A condition such as smoking that approximately doubles adult mortality has a much larger effect. We believe it would be highly desirable if quoted ratings (such as +100) or relative risks were routinely accompanied by the age range on which they are based. Of additional concern is the fact that RRs are critically dependent on which base population is used. For example, an RR of 2 based on a reference population of mostly males implies a much larger excess risk than if the reference population is mostly female.

Rating up, another commonly used method, in effect advances the individual's age by a certain number of years to reflect his medical condition. The method is seen to overestimate older-age mortality even more than the assumption of constant relative risk. In the comparisons reported here, the average underestimation of life expectancy was approximately 20% (see Table). The simplicity of rating up is an attractive feature (the general population life tables can still be used), but otherwise there seems little to recommend it.

Although less widely used than constant RR, the assumption of constant EDR has been recommended.[24] This is appropriate in some circumstances. In practice, however, the EDRs in most conditions do increase with age. The evidence indicates that constant EDR generally underestimates older-age mortality, and thus overestimates life expectancy. On average, the overestimation was 16% in the comparisons shown in the Table.

The assumption of log-linear declining relative risk (LDR) performed the best of the methods considered here. The average absolute error in the estimated life expectancies (ie, with sign ignored) was 2.0 years, and the bias (ie, the average error when signs are taken into account) was only 3%. The assumption has some theoretical basis. Mortality rates in the general population rise approximately exponentially, at least over the age range of 30–70 (Gompertz law). If mortality in the condition of interest also climbs exponentially even though with a different rate of increase, then as a mathematical consequence, LDR holds.

An issue arising in the application of LDR is the parity age, the age at which the relative risk associated with the condition declines to 1.0. We have found that parity at 100 is a reasonable choice for cerebral palsy (this is a refinement of our earlier work,[1] where parity at ages 85 to 95 were suggested) and for traumatic brain injury,[25] diabetes (Figure 3), and other conditions. However, spinal cord injury is an exception: the parity age is higher and may even be infinite (ie, constant, rather than declining, relative risk). Our suggestion is that LDR with parity age 100 may be a reasonable default choice in general unless there is evidence (or clinical argument) to the contrary, but further empirical comparisons here would be valuable.

The proportional life expectancy (PLE) method for estimating old-age mortality has not been previously proposed. Anderson[8,16] appears to have been the first to show that, for several medical conditions, the life expectancy expressed as a proportion of general population life expectancy is either constant or only slowly changing as age increases. This insight, combined with the mathematical relationship proved in the Appendix, leads to a usable estimation technique. According to the evidence presented here, the method does not perform quite as well as LDR, and it has some tendency to underestimate hazards at older ages. The graphs show, however, that the estimates derived from PLE are usually rather similar to those from LDR, and the method seems generally superior to the more common assumptions of constant RR, constant EDR or rating up. Further, PLE has the advantage over LDR of not requiring the assumption of a parity age. We have found the PLE method to be useful in empirical work, and believe it merits further consideration.

The methods we have considered are ap-

plicable to long-term hazards in ''static'' conditions, such as cerebral palsy, traumatic brain injury or spinal cord injury. They may be also reasonable for some chronic or slowly developing conditions, such as diabetes or coronary artery disease, although further investigation would be valuable here. Another possible application is to ''lifestyle'' factors, such as obesity or being sedentary. On the other hand, none of the methods we have considered are likely to be applicable to acute conditions, such as pulmonary embolisms or most forms of cancer. Neither would they be expected to apply to degenerative conditions (eg, multiple sclerosis, Parkinson disease). In this context it may be noted that, unlike cerebral palsy, Down syndrome should be considered a degenerative condition beyond the age of about 40, and indeed the data suggest that the relative risk in Down syndrome actually increases with age beyond 40.[1] Such a pattern is incompatible with all the methods we have considered. A common feature of these acute or degenerative conditions is that they may not be associated with the usual exponential rise in mortality with age, and therefore may not meet the LDR assumption.

It should be noted that the present study has focused on the extrapolation of mortality rates outside the age range where data is available. Other forms of extrapolation are also important in practice. A common problem arises in applying published data from epidemiological studies or clinical trials to an insured population. Suppose that in the clinical trial, the mortality rates of persons with and without the medical condition are *a* and *b*, respectively. To apply this to an insured population, should one assume an EDR of (a-b), an RR of a/b, or use some other adjustment?

Research data to guide this choice is limited. The issue has been recently studied by Roudebush and Klein,[26] who concluded that constant EDR was the best of the methods they considered. Note that the newer methods considered in the present article—LDR and PLE—are specific to extrapolation across *ages*, and thus are not applicable in this situation. For this reason and others, it is certainly not our intention to suggest that standard methods relying on constant RR or EDR are no longer useful.

We conclude with a caveat. Although we believe that the comparisons made here are sufficient to justify the broad conclusions discussed above, it must be acknowledged that empirical comparisons available so far are rather limited. A difficulty in evaluating the various methods is that there are relatively few chronic medical conditions where the true mortality rates over a wide range of ages can be estimated with precision. No doubt further empirical comparisons will lead to further insights. It is hoped that the present work at least represents a start in this direction.

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APPENDIX

Theorem: If PLE holds, then the excess death rates are inversely proportional to the remaining life expectancies.

Proof

As specified in the section on PLE, the fundamental hypothesis is that

$$e_c(t)/e(t) = r. \tag{1}$$

The remaining life expectancy at age t, e(t), can be written

$$e(t) = \int S(x)\,dx/S(t), \tag{2}$$

where the integral is from t to infinity, and thus

$$\ln\{e(t)\} = \ln\{I(t)\} - \ln\{S(t)\}, \tag{3}$$

where

$$I(t) = \int S(x)\,dx. \tag{4}$$

It follows that

$$\begin{aligned} d/dt[\ln\{e(t)\}] &= d/dt[I(t)]/I(t) + h(t) \\ &= -1/e(t) + h(t). \end{aligned} \tag{5}$$

Similarly,

$$d/dt[\ln\{e_c(t)\}] = -1/e_c(t) + h_c(t). \tag{6}$$

Now according to (1)

$$\ln\{e_c(t)\} = \ln\{e(t)\} + \ln\{r\} \tag{7}$$

so

$$d/dt\,\ln\{e_c(t)\} = d/dt\,\ln\{e(t)\}. \tag{8}$$

Thus, the right hand sides of (5) and (6) are equal. Hence

$$-1/[r \cdot e(t)] + h_c(t) = -1/e(t) + h(t), \quad (9)$$

and the excess death rates are

$$h_c(t) - h(t) = k/e(t), \quad (10)$$

where $k = (1 - r)/r$.

Comments

1. Note that the units for (10) are appropriately those of a rate per year.
2. As the remaining life expectancy is generally a decreasing function of t, the EDR is generally monotonically increasing in t. Hence, the PLE estimates of hazards at old ages are generally higher than those based on the assumption of constant EDR.
3. The rate of increase in $\{1/e(t)\}$ is generally slower than that of h(t). The PLE estimates of hazards are therefore generally lower than those based on constant RR.
4. To implement PLE all that is needed is a reliable estimate of $EDR(t_0)$ for some given age t_0. One then computes $e(t_0)/e(t)$ for all t, and uses $h_c(t) = h(t) + EDR(t_0) \cdot e(t_0)/e(t)$. The ratio of expectancies in this formula will generally be computed from data on the reference population.