Mathematical Population Studies
1999, Vol. 7(2), pp. 161–177
Reprints available directly from the publisher
Photocopying permitted by license only

A LONG PERIOD MULTISTATE LIFE TABLE USING MICRO DATA

ROBERT SHAVELLE* and DAVID STRAUSS

Department of Statistics, University of California, Riverside, CA 92521, USA

The multistate life table (MLT) has been widely used by demographers for the past twenty years. However, the pivotal Markov condition upon which the entire methodology rests is rarely satisfied in practice. We lessen reliance upon the assumption by computing transition probabilities for longer periods of time than was previously practical. An extended Kaplan–Meier estimator accomplishes this task, simultaneously addressing the issue of censoring. This allows for the construction of a long period MLT. We provide an illustrative example of a 10-year period MLT, with comparison to a 1-year period MLT.

KEY WORDS: Kaplan–Meier estimator; longitudinal data; micro data

INTRODUCTION

The multistate life table (MLT) has been widely used by demographers for the past twenty years. Keyfitz (1985) and Schoen (1988a) indicate a wide range of applications, including marital status and labor force changes.

The entire MLT methodology is predicated upon the Markov assumption: future transitions do not depend upon the past, given that the current state is known (Keyfitz, 1985; Manton and Stallard, 1984; Schoen, 1988a). This assumption, however, is nearly always violated in real applications. Few biological systems (Beck and Pauker, 1983) or any social processes (Hannan, 1984; Heckman and Singer, 1982; Singer and Spilerman, 1974; Tuma and Hannan, 1984) obey the Markov assumption; see also Courgeau and Lelievre (1992), Hoem and Jensen

*Corresponding author. Tel.: (909) 787 4631. Fax: (909) 787 3286.
E-mail: shavelle@citrus.ucr.edu.

(1982), and Schoen (1988a). Keyfitz (1980) considers it to be the main limitation to more widespread use of the MLT. Failure of the assumption can lead to serious errors, such as biased transition probabilities ("lumping on the diagonals," Blumen *et al.*, 1955; McGinnis, 1968) and incorrect life expectancies (Ledent, 1980a).

A distinction needs to be made between the use of transition *rates* and transition *probabilities*. The former are instantaneous forces of transition, even if computed from a period of one year or more. By contrast, the latter are probabilities of being in a given state at the end of a period, for any given original state. MLTs constructed from rates or probabilities are often referred to as the "movement" and "transition" approaches, or option 1 and option 2, respectively (Ledent, 1978; 1980a,b; Rogers, 1975). Ledent and others (Ledent, 1980a,b; Ledent and Rees, 1986) have noted that the latter ameliorates the Markov problem: the longer the period the fewer times the Markov assumption need be invoked.

The primary difficulty in the transition approach is how to account for withdrawal or loss of persons from the study population. The key innovation here is to use an extended Kaplan–Meier estimator which has recently become available (Strauss and Shavelle, 1998a). The estimator addresses the issue of censoring while computing consistent estimates of the multistate transition probabilities. This allows for the construction of a longer period MLT than has previously been practical.

In this paper we show how to implement this idea using event-history (longitudinal micro) data.[1] The next section details the theory of a long period multistate life table. Section 3 provides an illustrative example for a 10-year period, with comparison to the analogous 1-year period MLT. We close with a discussion of practical issues.

THEORY

The key to the construction of a long period MLT is estimation of transition probabilities over an arbitrary length of time. We therefore

[1] The existence of large longitudinal data sets in recent years has led to the development of event-history analysis – a rapidly growing subfield of demography (see, e.g., Hannan, 1984; Hobcraft and Murphy, 1986; or Menken *et al.*, 1981) – and a call for new methods to maximize the information therein (Hannan, 1984; Heckman and Singer, 1982; Keyfitz, 1987; Manton and Stallard, 1988; Nour and Suchindran, 1985). It has been postulated that the confluence of event-history analysis and multistate demography could produce a formidable analytic tool (Hannan, 1984; Schoen, 1988b). The present contribution is such a nexus, and might also aptly be termed a semi-longitudinal MLT.

first describe the extended Kaplan–Meier (EKM) estimator which can accomplish such. We then show how it may profitably be applied to MLT methodology.

The Extended Kaplan–Meier Estimator

Suppose that we have N-year event-history (longitudinal micro) data on a cohort of individuals (of various ages). Let the state space be $\{1, 2, \ldots, k, \delta\}$, i.e., there are k transient (live) states and one absorbing (dead) state. Let $\{Z(x) = j\}$ denote that the person is in state j at age x. Let i and j be any two transient states, not necessarily distinct, and define

$$\pi_{ij}(x, n) = Pr\{Z(x+n) = i \mid Z(x) = j\}, \quad \text{for } 0 \leq n \leq N. \tag{1}$$

Let

$$\hat{S}_j(x, n) \tag{2}$$

denote the usual Kaplan–Meier estimator of the probability of surviving n $(0 \leq n \leq N)$ additional years for those in state j at age x (Collett, 1994; Kalbfleisch and Prentice, 1980; Kaplan and Meier, 1958; Lee, 1992). The Kaplan–Meier estimator gives an unbiased estimate of the survival probability under the assumption of non-informative censoring.

Let $P_{ij}(x+n, n)$ be the number of people in state i at age $x+n$ who were in state j at age x. The EKM estimator is given by

$$\hat{\pi}_{ij}(x, n) = \frac{P_{ij}(x+n, n)}{\sum_{i=1}^{k} P_{ij}(x+n, n)} \hat{S}_j(x, n), \quad \text{for } 0 \leq n \leq N. \tag{3}$$

For a detailed derivation and discussion see Strauss and Shavelle (1998a).[2]

Equation (3) encompasses transitions from any live (transient) state j at age x to any other live state i at age $x+n$. The remaining possibility is to transfer to the dead state, δ. This probability may be found by subtracting the sum of the aforementioned values from (1),

[2] A comparison of the EKM estimator to the integrated survival curve (Manton and Soldo, 1985) is also given by Strauss and Shavelle (1998a). The estimator has been used in an application to prognosis for survival and improvement of children with developmental disabilities (Strauss *et al.*, 1997; Strauss and Shavelle, 1998c) and to facilitate computation of life expectancy for persons who have suffered a traumatic brain injury (Strauss and Shavelle, 1998b).

which results in the usual estimator

$$\hat{\pi}_{\delta j}(x,n) = 1 - \hat{S}_j(x,n). \tag{4}$$

We have thus established that multi-step transition probabilities may be computed directly from longitudinal micro data in the presence of censoring. The EKM estimator enables us to lessen reliance upon the Markov assumption when constructing a multistate life table. This follows because we may *directly* estimate any n-step ($n \leq N$) transition matrix, and need not separately estimate n 1-step transition matrices and use the Markov assumption $(n-1)$ times to justify their multiplication.[3]

A Long Period MLT

Following Keyfitz's (1985) notation, let $l_{ij}(x)$ be the probability that a person is in state i at age x given that he was in state j at birth. Let $\mathbf{l}(x)$ be the matrix whose (i,j)th element is $l_{ij}(x)$. All life table functions may be computed once the set of transition matrices from age 0, $\{\mathbf{l}(x), x = 1, 2, \ldots\}$ is known (Ledent, 1980a; Hoem and Jensen, 1982). As we are not following a cohort from birth until all have died, we must link together the estimated N-step transition matrices in order to completely specify all possible transitions. We do so using the Markov assumption at ages $N, 2N, \ldots$, which we subsequently refer to as nodes.

Our procedure for constructing a long period MLT is as follows:

- Determine the longest length of time, N, for which reliable transition probabilities may be estimated. In practice this entails checking that the loss of persons to death and censoring does not lead to sample sizes so small as to preclude estimation of the longer-step transitions. The length of the observation period, N, is allowed to be larger than the width of the age groups. Use of smaller-width age groups, so as to avoid aggregation bias (Rogers, 1995), will be possible if a large enough data set is available.
- For ages $x \leq N$ compute $\mathbf{l}(x)$ empirically using (3) and (3), i.e.,

$$\mathbf{l}(x) = \boldsymbol{\pi}(0,x) \quad \text{for } 0 \leq x \leq N, \tag{5}$$

where $\boldsymbol{\pi}(0,x)$ is a matrix with (i,j)th element $\pi_{ij}(0,x)$.

[3] The context of multi-step transition matrices here implies that n is a positive integer. The derivation of the estimator, however, was for n an arbitrary positive real number.

- For ages $x^* > N$, with $x^* = kN + m$ ($x^*, k, N, m \in \mathbb{N}$, $0 \leq m < N$), compute $\mathbf{l}(x^*)$ by making only *partial* reliance upon the Markov assumption as

$$\mathbf{l}(x^*) = \boldsymbol{\pi}(0, N) * \boldsymbol{\pi}(N, N) * \cdots * \boldsymbol{\pi}([k-1]N, N) * \boldsymbol{\pi}(kN, m). \quad (6)$$

  For example, with $N = 10$, we let $\mathbf{l}(15) = \boldsymbol{\pi}(0, 10)\boldsymbol{\pi}(10, 5)$.
- As in the usual multistate life table (Keyfitz, 1985), we compute $\{\mathbf{T}(x), x = 0, N, 2N, \ldots\}$ using

$$\mathbf{T}(x) = \int_x^\infty \mathbf{l}(u)\, \mathrm{d}u, \quad (7)$$

  where $t_{ij}(x)$ is the number of years live in state i by persons in state j at age x. We also compute $\{\mathbf{e}(x), x = 0, N, 2N, \ldots\}$ using

$$\mathbf{e}(x) = \mathbf{T}(x)[\mathbf{l}(x)]^{-1}, \quad (8)$$

  where $e_{ij}(x)$ is the expected time to be spent in state i by persons in state j at age x.

Note that estimation of transition probabilities for alternate durations via EKM ($\pi_{ij}(x, n)$, $0 < n \leq N$) allows for determination of single year $\mathbf{l}(x)$ values between all nodes of the long period MLT. This leads to more accurate results for $\mathbf{T}(x)$, and consequently $\mathbf{e}(x)$, than the customary (linear or cubic) interpolation between nodes N years apart.

Values of $\mathbf{e}(x)$ at the nodes are clearly interpretable as a result of the Markov assumption, and apply to a synthetic cohort followed forward in time. The single year $\mathbf{l}(x)$ values, together with Equations (7) and (8), will yield $\mathbf{e}(x)$ values between the nodes. The latter may be regarded as interpolates, as they do not correspond to any distinct synthetic cohort.

EXAMPLE

The application here concerns the prognosis for children with severe developmental disabilities. We now describe the data, document the failure of the Markov assumption, sketch the construction of 10-year and 1-year period MLTs, and compare 10-year period and 1-year period MLTs with respect to transition probabilities and remaining life expectancies.

The Data

Our data source is the Client Development Evaluation Report (hereafter, CDER; Department of Developmental Services, 1979), filled out approximately annually for all people receiving money or services from the State of California. The portion of the data base used here consisted of 1,231,786 records for 175,637 people observed in the period 1980–1994. The CDER data has been described elsewhere (Eyman *et al.*, 1993a,b; Strauss *et al.*, 1996).

Each CDER is an extensive report, consisting of 100 diagnostic and 66 evaluative items. The variable of interest in the present study is a nine-point mobility scale whose low end is "cannot lift head when lying on stomach" and whose high end is "assumes and maintains sitting position independently." The above cited studies, and others, have demonstrated that this variable is a key predictor of survival. Here, we focus on a simple dichotomy of can/cannot lift head when lying on stomach. The two states are henceforth referred to as the good and bad states.

Typical questions to be asked are (1) what are the chances of survival and the chances of improvement for a child of a given age who currently cannot lift his head?, and (2) what is the life expectation for such a child, and how much of it is expected to be lived in the more debilitated state?

Failure of the Markov Assumption

Figure 1 shows the age-specific probabilities of improving (moving to the good state) in the next year for clients currently in the bad state. The graph shows two curves, one each for those whose state one year previously was good or bad. The probability of improving is much better for those who were in the good state one year earlier. In this case the immediate past does indeed affect the chance of future transition, contradicting the Markov assumption.

Figure 2 shows the age-specific probabilities of dying in the next year for those currently in the bad state. Again, we see that the transition probability varies depending upon the past history of the individual. Analogous graphs for those currently in the good state, and for other past histories (not limited to just the most previous state) are not shown here. The implication from all the graphs is that the Markov assumption is seriously violated. We now show the effect of the violation on multi-step transition probabilities.

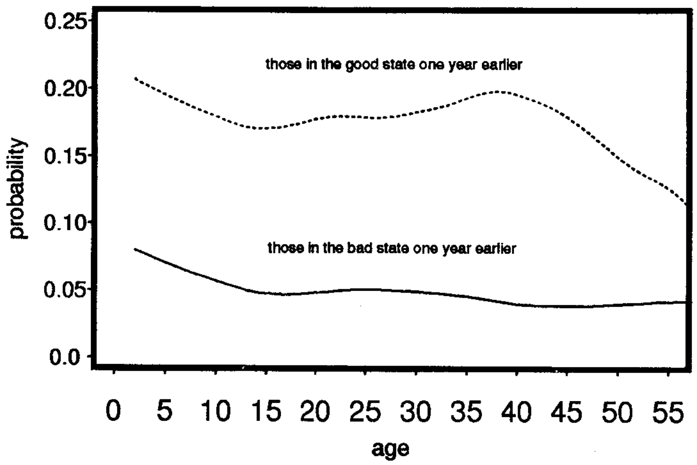

FIGURE 1 Probability of improving in the next year for persons currently in the bad state, stratified by state of occupancy one year earlier. The large disparity between the two curves is evidence that the Markov assumption fails.

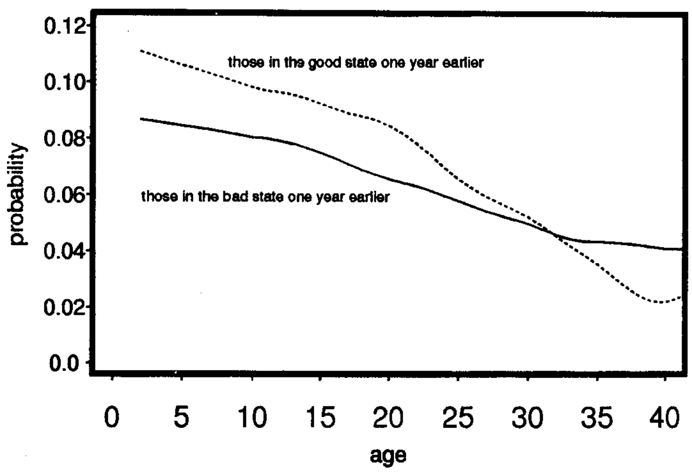

FIGURE 2 Probability of dying in the next year for persons currently in the bad state, stratified by state of occupancy one year earlier. The large disparity between the two curves is evidence that the Markov assumption fails. *Note*: the probability is higher for those previously in the good state because their condition is worsening, whereas those previously in the bad state have at least stabilized.

Construction of the 10-year and 1-year Period MLTs

Using the EKM estimator, we estimated $\pi_{ij}(x,n)$ for initial (rounded to the nearest year) ages[4] $x = 1, 10, 20, \ldots, 80$; lengths of time[5] $n = 1, 2, \ldots, 10$; initial states $j \in \{\text{bad}, \text{good}\}$; and destination states $i \in \{\text{bad}, \text{good}, \text{dead}\}$. The empirically computed values were smoothed to produce stable estimates of the transition probabilities.

Having estimated the 1- to 10-year transition matrices at each node, we computed the matrices $\{\mathbf{l}(x), x = 1, 2, 3, \ldots, 90\}$ using (5) and (6), with no persons surviving past age 90. Next, $\{\mathbf{T}(x), x = 1, 10, 20, \ldots, 80\}$ was computed using (7), with a linear approximation between single-year $\mathbf{l}(x)$ values. Lastly, the values of $\{\mathbf{e}(x), x = 1, 10, 20, \ldots, 80\}$ were computed using (8).

In addition, a 1-year period MLT was constructed using the transition approach from 1-year transition probabilities. The probabilities were those estimated by the EKM estimator (for intermediate $\mathbf{l}(x)$ values) for the 10-year period MLT. Values of $\mathbf{l}(x)$, $\mathbf{T}(x)$, and $\mathbf{e}(x)$ were computed using (5)–(8), but with nodes at single years of age. In this 1-year period MLT the Markov assumption is invoked 88 times, at ages $2, 3, \ldots, 89$; in the comparable 10-year period MLT above, 8 times, at ages $10, 20, \ldots, 80$.

Comparison to the Results of an MLT Obtained by Applying the Transition Method to 1-year Intervals

Tables 1 and 2 show the 10-year and 1-year period MLTs, respectively, for persons currently in the bad state. Columns 2–4 of each table show how 100,000 persons in the synthetic cohort at age 1 would be

---

[4] We have chosen to begin our analysis at age 1, rather than at birth, for several reasons. Firstly, precise measurement of the aforementioned variable is suspect at ages much younger than one year; in fact, there is no uniform age at which even healthy children may obtain full mobility skills. The assumption of "a cohort with no past" is thus valid here. Secondly, for reasons not germane to the present discussion, we doubt the assumption of non-informative censoring for those who leave the study population before age 1. Lastly, many developmentally disabled children only enter the system after age 3 months. Therefore, a full tracking from birth is not possible, and a later starting age is required.

[5] Although measurement of the variable of interest is made only yearly (panel data), the assumption that it is complete event-history data is sensible. Primarily, most children are evaluated very near their birthdays, so that little extrapolation from intermediate ages to integral ages is required. Additionally, clients are required by law to have an evaluation should their medical condition significantly alter, and a dramatic change in this variable would warrant such a reevaluation. Thus, if an observation at a particular integral age is unavailable, one may rightfully impute the value from the most recent prior CDER.

TABLE 1
Ten-year period MLT for persons currently in the bad state

Age	#Bad	#Good	#Dead	e	e (bad)	e (good)
1	100000	0	0	35.9	4.2	31.7
10	11440	51000	37560	21.1	9.0	12.1
20	4050	49170	46790	23.2	10.5	12.8
30	1790	46980	51230	26.6	12.6	14.0
40	1340	44120	54540	22.8	10.1	12.7
50	970	40250	58780	17.5	8.4	9.1
60	550	33560	65880	14.5	7.8	6.7
70	270	23130	76600	9.8	7.3	2.5
80	220	11900	87880	6.9	6.4	0.5

TABLE 2
One-year period MLT for persons currently in the bad state

Age	#Bad	#Good	#Dead	e	e (bad)	e (good)
1	100000	0	0	44.9	4.4	40.4
10	13065	59572	27361	34.8	8.1	26.6
20	4263	61433	34303	33.8	8.7	25.1
30	1781	60395	37823	29.9	9.5	20.4
40	1166	57824	41009	24.6	9.9	14.7
50	915	53951	45132	17.5	7.5	10.0
60	747	46602	52650	11.9	6.8	5.1
70	545	35499	63955	6.7	4.8	2.0
80	251	21884	77864	1.5	1.3	0.2

distributed amongst the three states at subsequent ages. Columns 5–7 give the total remaining life expectancy broken down by time to be spent in each state. The disparity between the two life expectancies is large at the younger ages (over 14 years at age 10), and decreases with age. As the 10-year period MLT relies less upon the questionable Markov assumption, its results are to be preferred to those of the 1-year period MLT.

Figure 3 compares MLT and EKM age-specific probabilities of transition from the bad to the good state. The 1-year transition probabilities for the two MLTs are identical (by construction), and thus are not shown. The 2- and 10-year transition probabilities for the 1-year period MLT were obtained by multiplying 1-year transition matrices under the Markov condition. The 2- and 10-year EKM transition probabilities were derived using the EKM estimator. The EKM estimated values are based *directly* on the longitudinal data, and

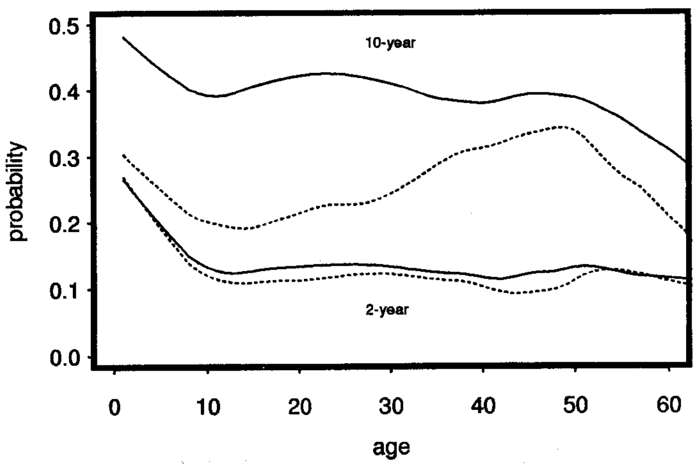

FIGURE 3 Comparison of MLT and EKM estimated 1-, 2-, and 10-year bad to good transition probabilities. Solid line (—): 1-year transition estimator. Dotted line (···): EKM estimator. The large difference between the two 10-year curves is due to the MLTs reliance upon the Markov assumption.

thus may be taken as "truth." This comparison of long-term estimated MLT transition probabilities to the "true" values appears to be unique to the literature.

We observe from Fig. 3 that the 2-year probabilities are similar for the age range shown, but that the 10-year values are markedly different up to about age 40. In fact, the probabilities are more than twice as large (40%) for the 1-year period MLT at age 10 as compared to the true (EKM) rate (20%). Because of the violation of the Markov assumption in this application, the 1-year period MLT greatly overestimates the long-term (10-year) probability of improving.

Figure 4 is a similar graph for age-specific probabilities of transition from the bad to the dead state. Again, the 2-year transition probabilities are similar, and the 10-year rates noticeably different, especially at the youngest ages. Because of the relation to the previous graph, we see that here the 1-year period MLT systematically underestimates the long-term probability of dying (and also of staying in the bad state).

We now compare expectancies, restricting out attention to total remaining life expectancies. Figure 5 shows results based on three different methods. The 1-year and 10-year period MLTs are as described above. The "basic life table for bad only" refers to a (scalar) life table constructed from mortality rates for those in the bad state

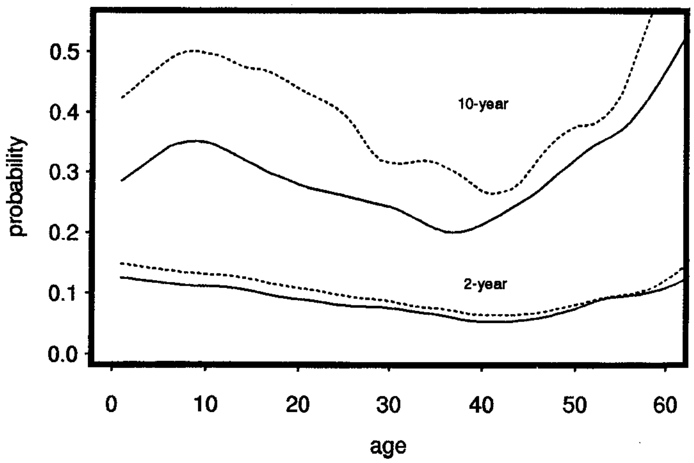

FIGURE 4 Comparison of MLT and EKM estimated 1-, 2-, and 10-year bad to dead transition probabilities. Solid line (—): 1-year transition estimator. Dotted line (···): EKM estimator. The difference between the curves is due to the MLTs reliance upon the Markov assumption.

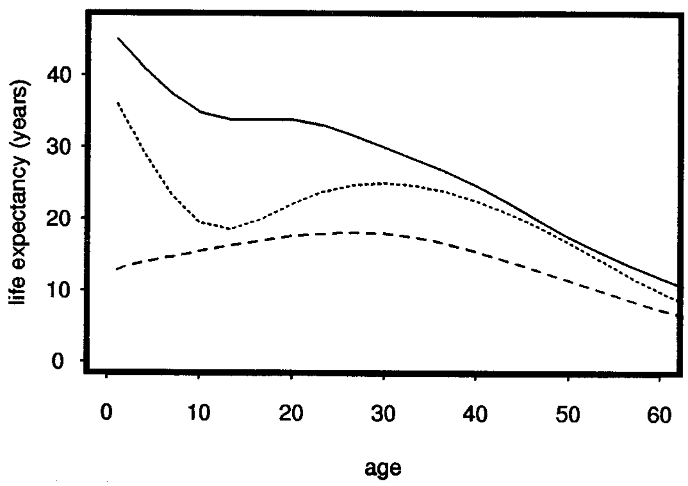

FIGURE 5 Comparison of residual life expectancies for persons currently in the bad state. Solid line (—): MLT based on 1-year transition estimator. Dotted line (···): MLT based on 10-year EKM estimator. Dashed line (---): basic life table for bad state only.

only. It gives life expectancies only for those currently in the bad state, assuming they remain there until death, and thus provides a useful lower bound.

The graph highlights several interesting items. First, the 1-year and 10-year period MLTs give very different answers up to about age 35, with the difference entirely due to the former's reliance upon the Markov assumption. Second, the results are similar from about age 40 onward, roughly paralleling the similarity of curves of Figs. 3 and 4 past age 40. Third, at ages 10–20, the 10-year period MLT values are much nearer to the "bad only" values than to those of the 1-year period MLT, indicating that persons in the bad state at those ages are much more likely to remain there until death than the 1-year period MLT indicates. This corresponds to the results of Fig. 3 for the same ages, where the long-term (10-year) chance to improve is actually much lower than the 1-year period MLT predicts. Last, the graph shows the danger in ignoring possible changes in state: the remaining life expectancies for those currently in the bad state are significantly different depending upon whether the possibility of improvement in condition is modelled (either MLT) or not ("life table for bad only").

DISCUSSION

In constructing an MLT, accurate estimation of transition probabilities is an issue, especially from small sample sizes. This is a concern in any practical application of demography, and in all empirical work. The field of statistics has a vast literature on this. In general, if the sample size is too small to profitably use the nonparametric Kaplan–Meier estimator, one may use a parametric model to estimate the survival function or the transition probabilities. This procedure is common; see, for example, Blossfeld *et al.* (1989); Coleman (1964a,b; 1968; 1981); Espenshade (1987); Ginsberg (1971, 1972); Heckman and Singer (1982); Hougaard (1984); Land *et al.* (1994); MacRae (1977); Manton and Stallard (1980); Manton *et al.* (1986); McFarland (1970); Singer and Spilerman (1974); or Spilerman (1972a,b).

As noted earlier, the major limitation of the MLT is its reliance upon the Markov assumption. With a long period MLT, however, multi-step transition probabilities computed using the EKM estimator are essentially empirical up to the length of the observation period, N, with no Markov assumption required. In regard to life expectancies, however, use of the Markov condition at the nodes is essential in order to use the multistate framework.

To completely bypass use of the Markov assumption it would be necessary to consider the entire past history at each age; that is, to use an event-history model (Hannan, 1984). Special cases include models with duration dependence (Belanger, 1989; Wolf, 1988) and semi-Markov processes (Ginsberg, 1971; 1979; Hennessey, 1980; Hoem, 1972; Kitsul and Philipov, 1982; Littman and Mode, 1977; Mode, 1976; 1982; Pyke 1961a,b). These are rarely used, however, as they are impractical and require age-duration-specific rates (Courgeau and Lelievre, 1992; Ginsberg, 1971; Schoen, 1988b).

If data is abundant, one could follow an age- and state-specific cohort with a given past history until all have died, and then compute (empirically) the transition probabilities and average time spent in the various states. This is akin to a purely longitudinal MLT (Hoem and Jensen, 1982; Willekens, 1987) with "birth" at age x – that is, all persons are followed from birth until death and the state of occupancy at each age noted. The results of such an endeavor, obtained many years later, would be of only historical interest. It might be argued, then, that even if full event-history data were available, use of the most recent N years of data by a long period MLT would provide more timely and appropriate results. Analogously, this is why the period life table is preferred to the cohort version (World Health Organization, 1984).

In summary, a long period MLT provides all the useful derived quantities of MLT methodology while ameliorating its main problem: reliance upon the Markov assumption. The increasing availability of longitudinal data sets should ensure many new applications.

ACKNOWLEDGEMENTS

The authors thank Robert Schoen for helpful comments on an earlier version of the manuscript. They also extend special thanks to the editor and two referees for many suggestions which clarified the issues and substantially improved the paper.

REFERENCES

Beck, J.R. and Pauker, S.G. (1983) The Markov process in medical prognosis. *Medical Decision Making* 3: 419–458.

Belanger, A. (1989) Multistate life table with duration dependence: An application to Hungarian female marital history. *European Journal of Population* 5: 347–372.

Blossfeld, H.P., Hamerle, A. and Mayer, K.U., (Eds.) (1989) *Event History Analysis.* Hillsdale, N.J.: Lawrence Erlbaum Associates, Inc.

Blumen, I., Kogan, M. and McCarthy, P.J. (1955) *The 1955 Cornell Studies of Industrial and Labor Relations 5. The Industrial Mobility of Labor as a Probability Process.* Ithaca: Cornell University Press.

Coleman, J.S. (1964a) *Introduction to Mathematical Sociology*. New York: Free Press.

Coleman, J.S. (1964b) *Models of Change and Response Uncertainty*. Englewood Cliffs, New Jersey: Prentice-Hall.

Coleman, J.S. (1968) The mathematical study of social change. In H.M. Blalock and A.B. Blalock (Eds.), *Methodology in Social Research*, pp. 428–478. New York: McGraw-Hill.

Coleman, J.S. (1981) *Longitudinal Data Analysis*. New York: Basic Books.

Collett, D. (1994) *Modelling Survival Data In Medical Research*. New York: Chapman & Hall.

Courgeau, D. and Lelievre, E. (1992) *Event History Analysis In Demography*. Oxford: Clarendon Press.

Department of Developmental Services (1979) Client Development Evaluation Report. Sacramento, CA: Author.

Espenshade, T.J. (1987) Marital careers of American women: A cohort life table analysis. In J. Bongaarts, T.K. Burch, and K.W. Wachter (Eds.), *Family Demography: Methods and their Application*, pp. 150–167. Oxford: Clarendon Press.

Eyman, R.K., Grossman, H.J., Chaney, R.H. and Call, T.L. (1993a) Survival of profoundly disabled people with severe mental retardation. *American Journal on Diseases of Children* **147**: 329–336.

Eyman, R.K., Olmstead, C.E., Grossman, H.J. and Call, T.L. (1993b) Mortality and the acquisition of basic skills by children and adults with severe disabilities. *American Journal on Diseases of Children* **147**: 216–222.

Ginsberg, R. (1971) Semi-Markov process and mobility. *Journal of Mathematical Sociology* **1**: 233–262.

Ginsberg, R. (1972) A critique of probabilistic models: Application of the semi-Markov model to migration. *Journal of Mathematical Sociology* **2**: 63–82.

Ginsberg, R. (1979) Timing and duration effects in residence histories and other longitudinal data II: Studies of duration effects in Norway, 1965–1971. *Regional Science and Urban Economics* **9**: 369–392.

Hannan, M.T. (1984) Multistate demography and event-history analysis. In A. Diekmann and P. Mitter (Eds.), *Stochastic Modelling of Social Processes*. New York: Academic Press.

Heckman, J.J. and Singer, B. (1982) Population heterogeneity in demographic models. In K.C. Land and A. Rogers (Eds.), *Multidimensional Mathematical Demography*, pp. 567–599. New York: Academic Press.

Hennessey, J.C. (1980) An age dependent, absorbing, semi-Markov model of work histories of the disabled. *Mathematical Biosciences* **51**: 283–304.

Hobcraft, J. and Murphy, M. (1986) Demographic event history analysis: A selective review. *Population Index* **52**: 3–27.

Hoem, J.M. (1972) Inhomogeneous semi-Markov processes, select actuarial tables, and duration dependence in demography. In T.N.E. Greville (Ed.), *Population Dynamics*, pp. 251–296. New York: Academic Press.

Hoem, J.M. and Jensen, U.F. (1982) Multistate life table methodology: A probabilistic critique. In K.C. Land and A. Rogers (Eds.), *Multidimensional Mathematical Demography*, pp. 155–264. New York: Academic Press.

Hougaard, P. (1984) Life table methods for heterogenous populations: Distributions describing the heterogeneity. *Biometrika* **71**: 75–83.

Kalbfleisch, J.D. and Prentice, R.L. (1980) *The Statistical Analysis of Failure Time Data*. New York: John Wiley & Sons.

Kaplan, E.L. and Meier, P. (1958) Nonparametric estimation from incomplete observations. *Journal of the American Statistical Association* **53**: 457–481.

Keyfitz, N. (1980) Multidimensionality in population analysis. In K. Schuessler (Ed.), *Sociological Methodology, 1980*, pp. 191–218. San Francisco: Jossey-Bass.

Keyfitz, N. (1985) *Applied Mathematical Demography* (2nd edition). New York: Springer-Verlag.

Keyfitz, N. (1987) Form and substance in family demography. In J. Bongaarts, T.K. Burch, and K.W. Wachter (Eds.), *Family Demography: Methods and their Application*, pp. 3–16. Oxford: Clarendon Press.

Kitsul, P. and Philipov, D. (1982) High- and low-intensity model of mobility. In K.C. Land and A. Rogers (Eds.), *Multidimensional Mathematical Demography*, pp. 505–534. New York: Academic Press.

Land, K.C., Guralnik, J.M. and Blazer, D.G. (1994) Estimating increment–decrement life tables with multiple covariates from panel data: The case of active life expectancy. *Demography* **31**: 297–319.

Ledent, J. (1978) Some methodological and empirical considerations in the construction of increment–decrement life tables, RM-78-25. Laxenbury, Austria: International Institute for Applied Systems Analysis.

Ledent, J. (1980a) An improved method for constructing increment–decrement life tables from the transition perspective. WP-80-00. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Ledent, J. (1980b) Multistate life tables: Movement versus transition perspectives. *Environment and Planning*, Series A, **12**: 533–562.

Ledent, J. and Rees, P. (1986) Life Tables. In A. Rogers and F.J. Willekens (Eds.), *Migration and Settlement: A Multiregional Comparative Study*. Dordrecht: Boston.

Lee, E.T. (1992) *Statistical Methods for Survival Data Analysis* (2nd edition). New York: Wiley.

Littman, G.S. and Mode, C.J. (1977) A non-Markovian stochastic model for the Taichung medical IUD experiment. *Mathematical Biosciences* **34**: 279–302.

MacRae, E.C. (1977) Estimation of time-varying Markov processes with aggregate data. *Econometrica* **45**: 183–198.

Manton, K.G. and Soldo, B.J. (1985) Dynamics of health changes in the oldest old: New perspectives and evidence. *Milbank Memorial Fund Quarterly* **63**: 206–285.

Manton, K.G. and Stallard, E. (1980) A stochastic compartment model representation of chronic disease dependence: Techniques for evaluating parameters of partially unobserved age inhomogeneous stochastic processes. *Theoretical Population Biology* **18**: 57–75.

Manton, K.G. and Stallard, E. (1984) *Recent Trends In Mortality Analysis*. New York: Academic Press.

Manton, K.G. and Stallard, E. (1988) *Chronic Disease Modelling: Measurement and Evaluation of the Risks of Chronic Disease Processes*. New York: Oxford University Press.

Manton, K.G., Stallard, E. and Vaupel, J.W. (1986) Alternative models for the heterogeneity of mortality risks among the aged. *Journal of the American Statistical Association* **81**: 635–644.

McFarland, D.D. (1970) Intra-generational social mobility as a Markov process: Including a time-stationary Markovian model that explains observed declines in mobility rates over time. *American Sociological Review* **35**: 463–476.

McGinnis, R. (1968) A stochastic model of social mobility. *American Sociological Review* **33**: 712–723.

Menken, J., Trussell, J., Stempel, D. and Babakol, O. (1981) Proportional hazards life table models: An illustrative analysis of sociodemographic influences on marriage dissolution in the United States. *Demography* **18**: 181–200.

Mode, C.J. (1976) On the calculation of the probability of current family-planning status in a cohort of women. *Mathematical Biosciences* **321**: 105–120.

Mode, C.J. (1982) Increment–decrement life tables and semi-Markovian processes from a sample path perspective. In K.C. Land and A. Rogers (Eds.), *Multidimensional Mathematical Demography*, pp. 535–565. New York: Academic Press.

Nour, E. and Suchindran, C.M. (1985) Multistate life tables: Theory and application. In P.K. Sen (Ed.), *Biostatistics: Statistics in Biomedical, Public Health and Environmental Sciences*, pp. 143–162. North Holland: Elsevier Science Publishers B.V.

Pyke, R. (1961a) Markov renewal processes: Definitions and preliminary properties. *Annals of Mathematical Statistics* **32**: 1231–1242.

Pyke, R. (1961b) Markov renewal processes with finitely many states. *Annals of Mathematical Statistics* **32**: 1243–1259.

Rogers, A. (1975) *Introduction to Multiregional Mathematical Demography*. New York: Wiley.

Rogers, A. (1995) *Multiregional Demography: Principles, Methods and Extensions*. New York: Wiley.

Schoen, R. (1988a) *Modelling Multigroup Populations*. New York: Plenum Press.

Schoen, R. (1988b) Practical uses of multistate population models. *Annual Review of Sociology* **14**: 341–361.

Singer, B. and Spilerman, S. (1974) Social mobility models for heterogeneous populations. In H.L. Costner (Ed.), *Sociological Methodology* 1973–1974, pp. 356–401. San Francisco: Jossey-Bass.

Spilerman, S. (1972a) Extensions of the mover–stayer model. *American Journal of Sociology* **78**: 599–626.

Spilerman, S. (1972b) The analysis of mobility processes by the introduction of independent variables into a Markov chain. *American Sociological Review* **37**: 277–294.

Strauss, D.J., Ashwal, S., Shavelle, R.M. and Eyman, R.K. (1997) Prognosis for survival and improvement of children with developmental disabilities. *Journal of Pediatrics* **131**: 712–717.

Strauss, D.J., Eyman, R.K. and Grossman, H.J. (1996) The prediction of mortality in children with severe mental retardation: The effect of placement. *American Journal of Public Health* **86**: 1422–1429.

Strauss, D.J. and Shavelle, R.M. (1998a) An extended Kaplan–Meier estimator and its applications. *Statistics in Medicine* **17**: 971–982.

Strauss, D.J. and Shavelle, R.M. (1998b) Long term survival of children and adolescents after traumatic brain injury. *Archives of Pediatric and Developmental Medicine* **79**: 1095–1100.

Strauss, D.J. and Shavelle, R.M. (1998c) Life expectancy of adults with cerebral palsy. *Developmental Medicine and Child Neurology* **40**: 369–375.

Tuma, N.B. and Hannan, M.T. (1984) *Social Dynamics: Models and Methods.* New York: Academic Press.

Willekens, F.J. (1987) The marital status life table. In J. Bongaarts, T.K. Burch, and K.W. Wachter (Eds.), *Family Demography: Methods and their Application.* Oxford: Clarendon Press.

Wolf, D. (1988) The multistate life table with duration dependence. *Mathematical Population Studies* **1**: 217–245.

World Health Organization (1984) The uses of epidemiology in the study of the elderly: Report of a WHO scientific group on the epidemiology of aging. Technical Report Series, No. 706. Geneva: Author.